Injury Potential of Ejection Seat Cushions

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The current MIL Specification dynamic model of the human body for the seated, spinal case, is used to determine how the dynamic characteristics of an ejection seat cushion influence the incidence of spinal injury during an ejection. The theory is compared with computer studies and shows reasonable agreement. A more sophisticated dynamic model of the human body may be needed before too much reliance can be placed upon the conclusions reached in this analysis. However, the general approach seems to be valid, and the conclusions are broadly consistent with operational experience, namely that a thin, soft cushion may slightly attenuate the spinal injury potential of an ejection seat, whereas a thick, stiffer cushion will magnify the injury potential.

Nomenclature

a = a damping constant; $= 2K_2/m_2$
$\bar{c}$ = ratio of damping to the critical value; $= a/2\omega$
$F(t)$ = $e^{-(w^2t/a)} \int e^{w^2t/a} f(t)dt$
$f(t)$ = normalized cushion force, as a function of time t; = force/m_2
I_n = $\int_n f(t)dt^n$
I_1 = $\int_0^{t_B} f(t)dt$, the cushion bottoming impulse
$2K_2$ = linear damper coefficient for the man-model illustrated in Fig. 1; = damper force/velocity
k_2 = linear spring stiffness for the man-model illustrated in Fig. 1; = force/deflection
m_2 = mass of the man-model illustrated in Fig. 1
t = time
t_B = time at which the cushion bottoms out, having reached a deflection δ_{1B}
y_c = vertical space ordinate of the seat pan
$\ddot{y}_c$ = d^2y_c/dt^2, the seat-pan acceleration
$\ddot{y}_{cB}$ = the seat-pan acceleration at the instant of cushion bottoming
α,β = constants in Eq. (7), linearization of Eq. (6), as indicated in Fig. 5
δ_1 = deflection (from zero strain) of the cushion model in Fig. 1
δ_{1B} = cushion deflection for "bottoming," i.e., the deflection at which the cushion becomes rigid
δ_2 = deflection of the man-model in Fig. 1
δ_{2B} = deflection of the man-model at the instant of cushion bottoming
δ_{20} = initial deflection of the man-model at time $t = 0$
η = $(1 - \bar{c})^{1/2}$
ξ = a parameter which defines the maximum response of Eq. (5) to a first-order impulse; $= \omega^2\delta_{2B}/\omega(\dot{\delta}_{2B} + \dot{\delta}_{1B})$
ω = angular undamped frequency of the man-model in Fig. 1; $= (k/m_2)^{1/2}$

Introduction

EJECTION seats have been used in aircraft since World War II, when speeds became high enough to prohibit the safe escape of crewmen through hatches. In selecting the seat catapult acceleration, early workers were guided by the pioneering work of Ruff,[1] who computed the dynamic characteristics of the human spine from measurements made with cadaver material, and from this deduced a maximum safe acceleration.

Needless to say, it is of extreme importance to accurately predict the effect of acceleration induced loads in the human body. As the flight envelopes of aircraft widen, it becomes increasingly difficult to eject crew members rapidly enough for them to clear the vertical fin at high dynamic pressures and to achieve sufficient height from low-altitude escapes. There is continual pressure to increase the impulse of rocket/catapult accelerators, therefore. On the other hand, there is little point in achieving a safe escape trajectory if the acceleration required is so great that the crewman is seriously injured by it.

Since the war, many attempts have been made to refine Ruff's original criteria, often by experiments with live human subjects or animals. Eband[2] presented a summary of much of this work, and attempted to define "tolerable acceleration" limits empirically, for both spinal and transverse body axes, on the basis of the data available. Wider-ranging surveys, covering such additional related topics as vibration tolerance, were given by Von Gierke and Goldman[3] in 1961 and by Von Gierke[4] in 1964.

During the period 1950–1960, concentration on the acquisition of experimental data caused the neglect of theoretical work, and even the repudiation by some workers of Ruff's original dynamic analysis. As a result, some physically absurd limitations were placed upon ejection seat designers, and product improvement was hampered. Perhaps the best example of this is contained in Ref. 5, where a maximum "rate of onset" (the first derivative of applied acceleration with respect to time) was specified.† Thus, the shorter the duration, the smaller the acceleration peak which could be tolerated!

During 1960, Von Gierke, Kornhauser[6]‡ and Payne[7,8]‡ independently advocated the use of lumped parameter dynamic models to explain and predict human tolerance to acceleration. The perspective of time has shown that others predated them, notably Latham,[9] but almost certainly other workers also, who could not obtain support for their views at the time they were presented. A rather good example is a paper by Hess,[10] written in 1956, which treats the human body as a distributed parameter system, and shows that rate of onset cannot be a relevant parameter by itself. This paper pre-dated the Ref. 5 specification allowables by several years.

It is easy to show that any deformation mode of a distributed parameter system can be replaced by a lumped parameter system of the same order. Kornhauser[11] showed that living animals were no exception to this rule, and its conse-

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† It is still possible to find new specifications calling for a maximum tolerable rate of onset. Several have recently been issued in the field of automotive crash injury research.

‡ References cited here are typical, but not the earliest.

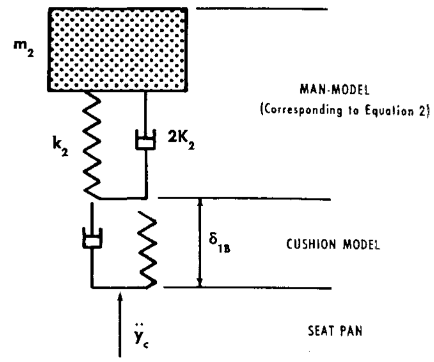

Fig. 1 Dynamic model of a man seated upon a cushion.

quences, by subjecting several hundred mice to accelerations of varying duration and magnitude, in order to prove that their injury and mortality rate corresponded to a particular value of the maximum deflection δ in the equation

$$\ddot{\delta} + \omega^2\delta = \ddot{y} \qquad (1)$$

where $\ddot{y}$ is the acceleration to which the animals were subjected, and δ and ω are the deflection and frequency of a hypothetical spring mass system representing a fundamental mode of deformation of the animal.

Payne[7] independently used the equation

$$\ddot{\delta} + a\dot{\delta} + \omega^2\delta = \ddot{y} \qquad (2)$$

and suggested that values for the damping coefficient a could be obtained from the mechanical impedance measurements of Coermann.[12] For the spinal mode of a seated man, Payne[13] showed that the lumped parameter man-model represented by Eq. (2) and illustrated in Fig. 1 corresponded to the mass of the upper torso and head, superimposed on the spring stiffness of the spine. The critical (injury-producing) value of $\omega^2\delta$, called the "Dynamic Response Index (DRI)," was deduced from several different types of experiment, and also by dynamic analysis of spine and upper torso as a structure, using cadaver data for stiffness and strength of the spinal elements (Stech and Payne[14]). The spinal man-model represented by Eq. (2) is now a standard tool in the design of ejection seats[15] and is found to give a reliable prediction of the spinal-injury potential of the various rocket/catapult accelerators used in aircraft. A comprehensive review of modern developments with this model has recently been given by Brinkley.[21]

The present paper uses this model to evaluate the dynamic effect of an ejection seat cushion, in order to determine its effect upon the model's DRI. Initial studies were carried out with the aid of an analog computer, using the linear spinal man-model illustrated in Fig. 1, in conjunction with various nonlinear cushion springs. The results obtained revealed no clear pattern, except that most cushions appeared to have an adverse effect on the dynamic response of the man-model. The need for a fairly fundamental theoretical analysis therefore became apparent.

Some earlier theoretical studies[13,16] employing linear analysis techniques had shown this approach to be fraught with complexity.§ Moreover, it was impossible to answer the question, "What is the best possible cushion?", because only linear models could be employed.

In the present investigation, the cushion restraint equations are studied in a more general way, in an effort to determine the basic phenomena and to discover whether protective cushions exist. It is first shown that the maximum deflection of the man-model must necessarily occur long after the cushion has bottomed out. It is therefore possible to separate the response of the man-model into two (additive) components: 1) the unknown response to the unknown seat-pan acceleration subsequent to cushion bottoming and 2) the response due to the seat-pan acceleration which occurred prior to cushion bottoming. Since the seat-pan acceleration is unknown, it is eliminated from the calculation of solution (2) by expressing the cushion force as an unknown function of time $[f(t)]$. A solution for the maximum model deflection in response to this excitation is then obtained. Since this solution is additive to solution (1), a positive response indicates an increase in the DRI.

Equations of Motion

The system to be investigated is illustrated in Fig. 1. The man-model (represented by a linear system) has a cushion in series with the input acceleration.¶ Without imposing constraints upon either the input acceleration or the nature of the cushion, we wish to discover what type of cushion will minimize the dynamic response of the man-model.

Let

$a = 2k_2/m_2$ (the damping constant), 1/sec

$\omega^2 = k_2/m_2$ (the spring constant), 1/sec^2

$f(t)$ = cushion force$/m_2$, ft/sec^2

The only mathematical constraint required of the function $f(t)$ is the physically inescapable one that it shall be sectionally continuous.

Prior to bottoming of the cushion, the equations of motion are[13]

$$\ddot{\delta}_2 + a\dot{\delta}_2 + \omega^2\delta_2 = \ddot{y}_3 - \ddot{\delta}_1 \qquad (3)$$

$$a\dot{\delta}_2 + \omega^2\delta_2 = f(t) \qquad (4)$$

After the cushion has bottomed out, δ_1 is constant, so that $\ddot{\delta}_1 = 0$, and the equation of motion is simply

$$\ddot{\delta}_2 + s\dot{\delta}_2 + \omega^2\delta_2 = \ddot{y}_c \qquad (5)$$

The influence of the cushion upon the maximum deflection of the man-model must necessarily appear in the *postbottoming phase* described by Eq. (5). As indicated in Fig. 2, the presence of the cushion reduces the values of δ_2 and $\dot{\delta}_2$ immediately prior to bottoming, but at the instant of bottoming introduces a velocity change spike of magnitude

$$\int_0^{t_B} \ddot{\delta}_1 dt$$

The impulsive velocity change associated with cushion bottoming will obviously be adverse, in that it will increase

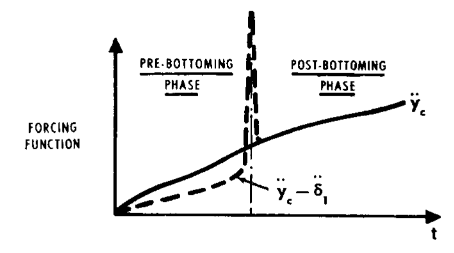

Fig. 2 Modification of the man-model forcing function due to the presence of a cushion.

§ The mathematical treatment was elementary, but the solutions were cumbersome and did not simply reveal the underlying physical picture.

¶ It is currently thought that a more elaborate model, in which a "pelvic mass" is inserted between the man-model and the cushion, would be more realistic, and more in accordance with steady-state vibration observations of human subjects.[17] Time considerations excluded the possibility of examining such a model during the present investigation, however.

the dynamic overshoot of the man-model in the postbottoming phase. The concomitant reduction in the values of δ_2 and $\dot{\delta}_2$ at the time of bottoming, due to the fact that $(\ddot{y}_3 - \dot{\delta}_1) < \ddot{y}_3$, is favorable, however, and the net effect depends upon the relative magnitudes of the two conflicting effects.**

As indicated in Fig. 3, the response of a system to the input of Fig. 3a can be duplicated by using the input of Fig. 3b, so long as the appropriate initial conditions are used in the latter case. These initial conditions are of course the values δ_{2B} and $\dot{\delta}_{2B}$ of δ_2 and $\dot{\delta}_2$, respectively, developed during the period $0 < t < t_B$.

As shown in Fig. 4, the excitation can be divided into two components. The stepped ramplike function $\Delta_1\ddot{y}_c$ is independent of whether or not a cushion is used, or its dynamic characteristics. The second-order impulse $\Delta_2\ddot{y}_c$ contains information about the previous history ($t < t_B$) of the system and the velocity change due to cushion bottoming. Changes in the cushion dynamics will therefore change $\Delta_2\ddot{y}_c$.

We define a "good" cushion as one which reduces the system's maximum response to $\Delta_2\ddot{y}_c$. This is not rigorously correct because of the differences in phase between the responses to $\Delta_1\ddot{y}_c$ and $\Delta_2\ddot{y}_c$. But since $\Delta_1\ddot{y}_c$ is unknown, and can vary randomly for even a particular escape system (together with the initial conditions), any attempt to account for phase effects is felt to be impractical. Additionally, for reasons explained later, phase attenuation for the practically important ramp excitation is likely to be quite small in practice.

The maximax response to the $\Delta_2\ddot{y}_c$ excitation is given in Ref. 20 as

$$\xi_{\max} = e^{-(\bar{c}/\eta)(\varphi-\theta)}[1 + 2\bar{c}\xi + \xi^2]^{1/2} \tag{6}$$

where

$$\xi_{\max} = \frac{\omega^2\delta_{2\max}}{\omega(\dot{\delta}_{2B} + \dot{\delta}_{1B})},\ \xi = \frac{\omega^2\delta_{2B}}{\omega(\dot{\delta}_{2B} + \dot{\delta}_{1B})}$$

$$\bar{c} = c/\omega = a/2\omega,\ \eta^2 = 1 - \bar{c}^2$$

$$\sin\theta = (\xi\eta)/[1 + 2\bar{c}\xi + \xi^2]^{1/2},\ \sin\varphi = \eta$$

The function $\xi_{\max} = f(\xi)$ is plotted in Fig. 5a. For analytical convenience, we define a linear approximation

$$\xi_{\max} = \alpha + \beta\xi \tag{7}$$

when the change in ξ is small. The values of β/α are plotted in Fig. 5b.

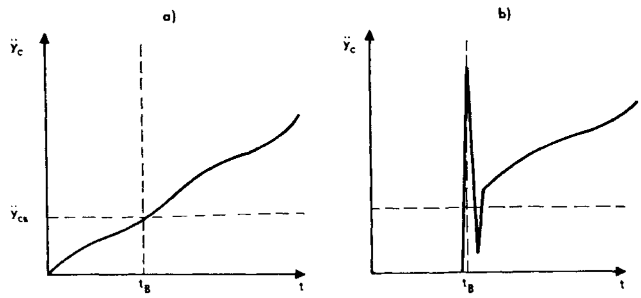

Fig. 3 Two forcing functions which give the same postbottoming response.

** Of course, one can conceive of a cushion whose stiffness and damping are comparable with the man-model, so that "bottoming-out" need not occur. Such a cushion would be uncomfortably rigid, and would always magnify the *total* response to finite rise-time excitations, because of the lower frequency of the cushion/man system, although not necessarily the response of the man-model itself. Needless to say, comfort requirements would prevent such a cushion being used in practice.

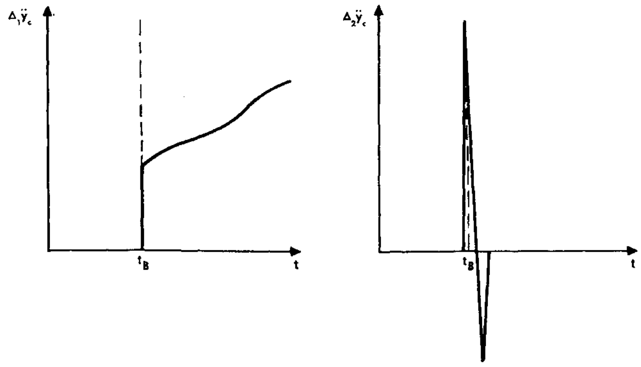

Fig. 4 Division of the forcing function into two separate components.

A Solution of the Equations of Motion

Integrating Eq. (4) from $t = 0$ to $t = t_B$, we obtain

$$\omega^2\delta_{2B} = \omega^2\delta_{20}e^{-(w^2t/a)} + (\omega^2/a)\,F(t) \tag{8}$$

where

$$F(t) = e^{-(w^2t_B/a)}\int e^{-(w^2t_B/a)}f(t)dt$$

Also, by substituting (4) in (3), and integrating from $t = 0$ to $t = t_B$,

$$\dot{\delta}_{1B} + \dot{\delta}_{2B} = \dot{y}_{cB} - \int_0^{t_B} f(t)dt \tag{9}$$

By using Eqs. (8) and (9) to evaluate ξ, and substituting in (7)

$$\omega^2\delta_{2\max} = \alpha\omega[\dot{y}_{cB} - \int_0^{t_B} f(t)dt] + \beta[\omega^2\delta_{20}e^{-(w^2t_B{}^1/a)} + \omega^2/a\,F(t)] \tag{10}$$

Differentiating (see Appendix) with respect to

$$I_1 = \int_0^{t_B} f(t)dt$$

$$\frac{d}{dI_1}(\omega^2\delta_{2\max}) = -\alpha\omega + \frac{\beta\omega^2}{a}\left[1 - \frac{\omega^2}{a}\frac{F(t)}{f(t)}\right] \tag{11}$$

Since the bottoming impulse I_1 is reduced by a cushion, relative to the case of no cushion, its effect is favorable if $d/dI_1 \times (\omega^2\delta_{2\max})$ is positive, i.e.,

$$\frac{a}{\omega} < \frac{\beta}{\alpha}\left[1 - \frac{\omega^2}{a}\frac{F(t)}{f(t)}\right] \tag{12}$$

i.e.,

$$\omega\frac{F(t)}{f(t)} < \frac{a}{\omega}\left[1 - \frac{a}{\omega}\frac{\alpha}{\beta}\right] \tag{13}$$

Taking the value $a/\omega = 2\bar{c} = 0.6$ from the dynamic model of Ref. 15, it can be seen from Fig. 5 that

$$\left[1 - \frac{a}{\omega}\frac{\alpha}{\beta}\right] \geqslant 0.56,\ 0.6 > \frac{a}{\omega}\left[1 - \frac{a}{\omega}\frac{\alpha}{\beta}\right] \geqslant 0.264 \tag{14}$$

The ratio α/β varies with ξ, which is given by

$$\xi = \frac{\omega^2\delta_{20}e^{-(w^2t_B/a)} + (\omega^2/a)\,F(t)}{\omega(\dot{y}_{cB} - I_1)} \tag{15}$$

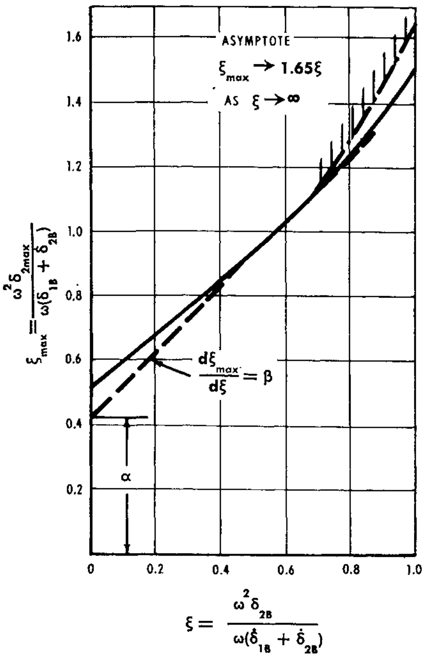

Fig. 5a Generalized response of a man-model to a second-order impulse ($\bar{c}$ = 0.3) as given by Eq. (6).

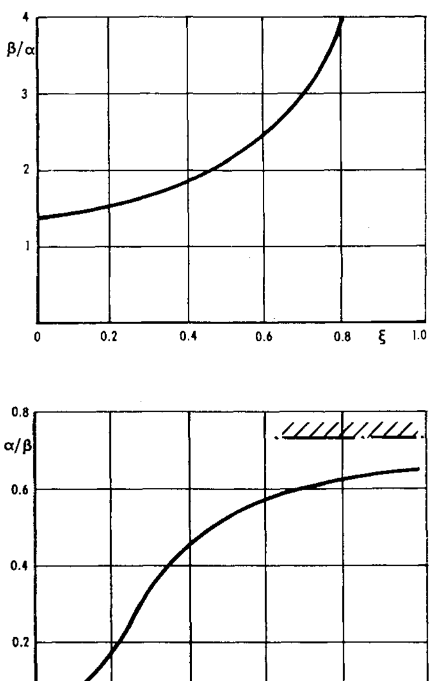

Fig. 5b Equivalent linear coefficients in the approximation $\xi_{max} = \alpha + \beta\xi$.

However, we know that in the initial phases of the system's response

$$\omega^2\delta_2 < a\dot{\delta}_2$$

$$\therefore \omega^2\delta_2 \ll \omega\dot{\delta}_2; \quad \omega = 0(10^2),\ a = 0(10^0)$$

$$\therefore a/\omega[1 - (a/\omega)\alpha/\beta] \simeq 0.264$$

Thus, the condition for cushion protection of the spinal model described in Ref. 15 (ω = 52.9 rad/sec) is

$$F(t_B)/f(t_B) < 0.264/52.9 = 0.005 \qquad (16)$$

Table 1 Analog data from Ref. 18, for a spinal model on various nonlinear cushions

Cushion	$m_2f(t_B)$, lb	$m_2\int_{\delta_{10}}^{\delta_{1B}} f(t_B)d\delta_1$, lb-ft	$\int_{\delta_{10}}^{\delta_{1B}} \frac{f(t)d\delta_1}{f(t_B)}$, ft	DRI/DRI$_0$
1	690	48.3	0.07	0.98
2	690	41.4	0.06	0.97
3	690	17.0	0.03	0.87
4	690	40.8	0.06	0.89
5	690	24.2	0.04	0.89
6A	690	73.8	0.11	1.03
6	690	85.8	0.12	1.15
7	690	96.1	0.14	1.11
8	690	149.9	0.22	1.52
9	640	24.0	0.04	0.94
10	640	65.6	0.10	0.99
11	640	110.4	0.17	1.00
12	640	152.2	0.24	1.25
13	690	58.5	0.09	1.00
14	690	122.4	0.18	1.04
15	690	130.8	0.19	1.05
16	690	105.6	0.15	1.15
17	690	196.0	0.28	1.15
18	690	132.4	0.19	1.14

or approximately††

$$\int_0^{t_B} f(t)dt/f(t_B) < 0.005 \qquad (17)$$

For $f(t)$ = const‡‡ (crushable foam), $t_{B\mathrm{crit}}$ = 0.005 sec. For $f(t) = k_1t$, $t_{B\mathrm{crit}}$ = 0.01 sec. These times are a very small fraction of the man-model's natural period of 0.1188 sec, justifying the foregoing simplification of $\omega^2\delta_2 \ll \omega\dot{\delta}_2$. The impulsive velocity change associated with cushion bottoming at the critical time is also very small; of the order of 0.2 fps. Thus, although a weak cushion will reduce dynamic overshoot and can be optimized for maximum effectiveness, the benefits of so doing will be fairly small.

When a substantial initial deflection or preload exists, the parameter ξ will be increased. But Eq. (14) shows that the parameter $a/\omega[1 - (a/\omega)\alpha/\beta]$ cannot in any case exceed about twice the small perturbation value. Thus, the foregoing conclusions can be taken to apply without restriction to small perturbations.

Comparison with Analog Studies

In Ref. 18, Shaffer has described the use of an analog computer to study Eqs. (3–5), using the spinal man-model and 19 different (nonlinear) experimental cushion force-deflection curves, driven by a typical ejection seat acceleration-time history.

It is not possible to correlate these results with the parameter $F(t_B)/f(t_B)$, since no record of $f(t)$ was made. However, as noted previously

$$\frac{F(t_B)}{f(t_B)} \simeq \int_0^{t_B} \frac{f(t)dt}{f(t_B)} = \frac{1}{f(t_B)} \int_{\delta_{10}}^{\delta_{1B}} \frac{f(t)d\delta_1}{(d\delta_1/dt)} \qquad (18)$$

For purposes of obtaining a *very rough* correlation, assume that $d\delta_1/dt$ may be regarded as a constant; so we obtain, as

†† For a discussion of the exponential integral, see Appendix.

‡‡ $m_2 \int_0^{t_B} f(t)dt$ is equal to the momentum which can be absorbed by a cushion (lb-sec). Thus, conventional pendulum tests of a cushion would be very suitable, were it not for the fact that the velocity-time history of a pendulum, mass is opposite to that experienced by a cushion during an ejection.

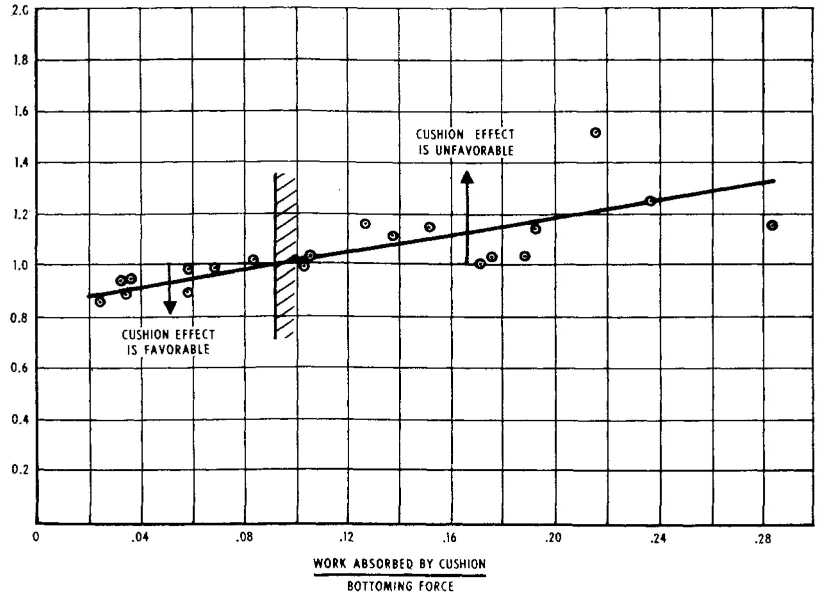

Fig. 6 Dynamic Response Index (DRI) magnification, as a function of an approximate cushion factor of merit.

an approximate factor of merit for a cushion,

$$\int_{\delta_{10}}^{\delta_{1B}} \frac{f(t)d\delta_1}{f(t_B)} = \int_{\delta_{10}}^{\delta_{1B}} \frac{\text{(cushion force)}}{\text{(bottoming force)}d\delta_1} \tag{19}$$

This parameter can be evaluated immediately from the curve of cushion force against deflection δ_1. Correlation of the parameter

$$\mathrm{DRI}/\mathrm{DRI}_0 = \omega^2\delta_{2\max} \text{ with cushion}/\omega^2\delta_{2\max} \text{ without cushion}$$

with Eq. (19) is quite good, as shown in Table 1 and Fig. 6. Because of the rather gross assumptions made to obtain Eq. (19), it is unrealistic to expect precise correlation, of course.

Attenuation by Phase Shift

A number of fundamental forcing functions can be obtained by successively differentiating the ramp function in Fig. 7a. Since a bottoming cushion, introducing forcing functions of the type shown in parts b and c will cause a maximum response to occur between $\pi/2$ and π, we might expect that this will subtract from the ramp response near $3\pi/2$. For this to be significant, however, the impulsive velocity change would have to be significant. Even if bottoming occurred at $\pi/2$, the impulse velocity change would be less than 2.0 fps, and this would not change the maximum response to the ramp because of the delay. If the cushion bottoming occurred before $\pi/2$, its solution could subtract from the ramp maximum, but the effect would be small because of the small velocity change. It is therefore concluded that a cushion's ability to change the phase of the dynamic system's response is of little practical utility.

However, it is interesting to note that the foregoing arguments can be applied to the acceleration profile of a catapult with considerable benefit. Payne and Shaffer[19] have shown that a velocity spike at the beginning of a conventional ramp acceleration input gives a 56% increase in seat velocity and 168% increase in seat travel without increase in the seat occupant's DRI. Even better results are shown to be possible with more complex acceleration profiles.

Conclusions

1) Within the variability inherent in operational systems, the phase shift effects of a cushion probably cannot be used to reduce the maximax response to ramp excitation, by any significant amount.

2) The parameter

$$\omega F(t_B)/(t_B) = e^{-(w^2t_B/a)} \int_0^{t_B} e^{w^2t/a} f(t)dt/f(t_B)$$

defines whether a given cushion [whose force $m_2f(t)$ is a function of time] will attenuate or increase the DRI. For thin cushions this parameter is approximately equal to

$$\frac{\omega I_1}{f(t_B)} = \omega m_2 \int_0^{t_B} \frac{f(t)dt}{f(t_B)} = \frac{\omega\cdot\text{(bottoming impulse)}}{\text{bottoming force}}$$

3) As a *very rough* guide, the parameter

$$\int_{\delta_{10}}^{\delta_{1B}} \frac{\text{(cushion force)}d\delta_1}{\text{bottoming force}} = \frac{\text{work absorbed in ft lb}}{\text{bottoming force in lb}}$$

should not exceed 1 ft if DRI magnification is to be avoided. This conclusion is consistent with the empirically observed high injury potential of the thick rubber foam cushions used in the early days of ejection seats.

4) Cushion bottoming impulse is a significant performance parameter. It should be measured for a velocity $\dot{\delta}_1 = k_nt^n$, however, rather than with $\dot{\delta}_1 = \dot{\delta}_{10} - k_nt^n$, which is characteristic of conventional pendulum tests.

5) The need exists for a more sophisticated study in which a lower "pelvic mass" is included in the dynamic model of the man, in order to see whether these conclusions need modification.

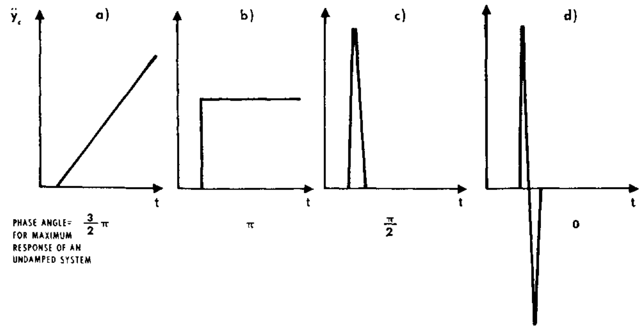

Fig. 7 Response of an undamped system to various acceleration inputs.

Appendix: Series Expansions of the Exponential Integral

Integration by parts gives

$$\int e^{pt}f(t)dt = e^{pt}[I_1 - pI_2 + p^2I_3 - \ldots] = e^{pt}\sum_{n=1}^{\infty}(-p)^{n-1}I_n$$

where

$$I_n = \int_n f(t)dt^n$$

$$F(t) \equiv e^{-(w^2t/a)}\int e^{w^2t/a}f(t)dt$$

$$= \sum_{n=1}^{\infty}\left(-\frac{\omega^2}{a}\right)^{n-1}I_n$$

$$\frac{dF(t)}{dI_1} = \sum_{n=1}^{\infty}\left[\left(-\frac{\omega^2}{a}\right)^{n-1}\frac{dI_n/dt}{dI_1/dt}\right]$$

$$= \frac{1}{f(t)}\sum_{n=1}^{\infty}\left(-\frac{\omega^2}{a}\right)^{n-1}I_{n-1} \quad \text{as } \frac{dI_n}{dt} = I_{n-1}$$

$$\text{and } I_0 = f(t)$$

$$= \frac{1}{f(t)}\left[I_0 + \frac{\omega^2}{a}\sum_{(n-1)=1}^{\infty}\left(-\frac{\omega^2}{a}\right)^{n-2}I_{n-1}\right]$$

$$= 1 + \frac{\omega^2}{a}\frac{F(t)}{f(t)}$$

Thus, for small values of ωt, $F(t) \approx I_1$. For example, the integral

$$e^{-at}\int^t e^{at}ktdt = k\left[\frac{t^2}{2!} - \frac{at^3}{3!} + \frac{a^2t^4}{4!} - \ldots\right]$$

$$\rightarrow kt^2/2 \text{ as at} \rightarrow 0$$

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